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INTERNATIONAL JOURNAL OF CURRENT RESEARCH

International Journal of Current Research
Vol. 4, Issue, 12, pp. 249-257, December, 2012

ISSN: 0975-833X

RESEARCH ARTICLE

PROPAGATION OF STONELEY, RAYLIEGH AND LOVE WAVES IN VISCOELASTIC MEDIA OF HIGHER ORDER

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ARTICLE INFO

Article History:
Received 12th September, 2012
Received in revised form
24th October, 2012
Accepted 25th November, 2012
Published online 18th December, 2012

Key words:
Rayleigh, Love Waves,
Viscoelastic solid medium

ABSTRACT

This paper describes a theory of surface waves propagating in a non-homogeneous, isotropic, viscoelastic solid medium of nth order including time rate of strain. The theory of generalized surface waves has been employed to investigate particular cases of Rayleigh wave, Love wave and Stoneley wave. When viscous field is neglected, the wave velocity equation of this generalized type of surface waves is in complete agreement with the corresponding classical results.

INTRODUCTION

When seismic waves propagate underground, they are influenced not only by the anisotropy of the media, but also by intrinsic viscosity of media given by Carcione [1]. Therefore, in order to accurately describe the underground propagation of the seismic waves and then more precisely guide seismic data acquisition, processing and interpretation, media models should be chosen that can simultaneously imitate anisotropic characteristics of formation and viscoelastic characteristics for numerical simulation and analysis of wave fields As a result, the theory of surface waves has been developed by Stoneley, Bullen, Ewing et al., Hunters and Jeffreys. The effect of gravity on wave propagation in an elastic solid medium was first considered by Bromwich [2], treating the force of gravity as a type of body force. Love [3] extended the work of Bromwich investigated the influence of gravity on superfacial waves and showed that the Rayleigh wave velocity is affected by the gravity field. Sezawa [4] studied the dispersion of elastic waves propagated on curved surfaces.

The transmission of elastic waves through a stratified solid medium was studied by Thomson [5]. Haskell [6] studied the dispersion of surface waves in multilayered media. A source on elastic waves is the monograph of Ewing, Jardtezky and Press [7]. Biot [8] studied the influence of gravity on Rayleigh waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible. Taking into account, the effect of initial stresses and using Biot's theory of incremental deformations, Dey modified the work of Jones [9]. De and Sengupta [10] studied many problems of elastic waves and vibrations under the influence of gravity field. Sengupta and Acharya [11] studied the influence of gravity on the propagation of waves in a thermoelastic layer. Brunelle [12] studied the surface wave propagation under initial tension of compression. Wave propagation in a thin two-layered laminated medium with stress couples under initial stresses was studied by Roy [13]. Datta [14] studied the effect of gravity on Rayleigh wave propagation in a homogeneous, isotropic elastic solid medium. Goda [15] studied the effect of inhomongeneity and anisotropy on Stoneley waves. Recently Abd-Alla and Ahmed [16] studied the Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress. In this work, the problem of nth order viscoelastic surface waves involving time rate of strain, the medium being isotropic and non-homogeneous has been studied. Biot's theory of incremental deformations has been used to obtain the wave velocity equation for Stoneley, Rayleigh and love waves. Further these equations are in complete agreement with the corresponding classical results in the absence of viscosity and non-homogeneity of the material medium.

2. Formulation of the problem

Let M_1 and M_2 be two non-homogeneous, viscoelastic, isotropic, semi-finite media. They are perfectly welded in contact to prevent any relative motion or sliding before and after the disturbances and that the continuity of displacement, stress etc. hold good across the common boundary surface. Further the mechanical properties of M_1 are different from those of M_2. These media extend to an infinite great distance from the origin and are separated by a plane horizontal boundary and M_2 is to be taken above M_1. Let Oxyz be a set of orthogonal Cartesian co-ordinates and let O be the any point on the plane boundary and Oz points vertically downward

to the medium M_i. We consider the possibility of a type of wave travelling in the direction Ox, in such a manner that the disturbance is largely confined to the neighborhood of the boundary which implies that wave is a surface wave. It is assume that at any instant, all particles in any line parallel to Oy having equal displacement and all partial derivatives with respect to y are zero. Further let us assume that u, □, w are the components of displacements at any point (x, y, z) at any time t.
The dynamical equations of motion for three-dimensional non-homogeneous, isotropic, viscoelastic solid medium in Cartesian co-ordinates are

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{12}}{\partial y} + \frac{\partial \tau_{13}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{1}$$

$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} = \rho \frac{\partial^2 \nu}{\partial t^2}, \tag{2}$$

$$\frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{23}}{\partial y} + \frac{\partial \tau_{33}}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}. \tag{3}$$

Where □□be the density of the material medium and $\square_{ij} = \square_{ji}$ ∀ i, j are the stress components.
The stress-strain relations for general isotropic, visco-elastic medium according to voigt are

$$\square_{ij} = D_{\square}\square\square\square\square_{ij} + 2\, D_{\square}\square e_{ij} \tag{4}$$

Where, $D_{\square}$, $D_{\square}$□are the functions of z and are elastic constants

$$\text{and } \square\square = \frac{\partial u}{\partial x} + \frac{\partial \nu}{\partial y} + \frac{\partial w}{\partial z} \tag{5}$$

Introducing eq (4) in (1), (2), (3), we get

$$D_\lambda \frac{\partial \Delta}{\partial x} + \Delta \frac{\partial D_\lambda}{\partial x} + 2D_\mu \frac{\partial^2 u}{\partial x^2} + 2\frac{\partial u}{\partial x}\frac{\partial D_\mu}{\partial x} + D_\mu \frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right] + \left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right]\frac{\partial D_\mu}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{6}$$

$$D_{\square}\square\square^2\ \square\square + \frac{\partial \nu}{\partial x}\frac{\partial D_\mu}{\partial x} + \frac{\partial \nu}{\partial z}\frac{\partial D_\mu}{\partial z} = \rho \frac{\partial^2 \nu}{\partial t^2}, \tag{7}$$

$$D_\mu \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\frac{\partial D_\mu}{\partial x} + D_\lambda \frac{\partial \Delta}{\partial z} + \Delta \frac{\partial D_\lambda}{\partial z} + 2D_\mu \frac{\partial^2 w}{\partial z^2} + 2\frac{\partial w}{\partial z}\frac{\partial D_\mu}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}. \tag{8}$$

We assume that the non-homogeneities for the media M_1 and M_2 are given by

$$D_{\square} = \sum_{K=0}^{n} \lambda_K e^{mz} \frac{\partial^K}{\partial t^K},\ D_{\square} = \sum_{K=0}^{n} \mu_K e^{mz} \frac{\partial^K}{\partial t^K},\ \square\square = \square_0 e^{mz} \tag{9}$$

$$D'_{\square\square\square} = \sum_{K=0}^{n} \lambda'_K e^{lz} \frac{\partial^K}{\partial t^K},\ D'_{\square} = \sum_{K=0}^{n} \mu'_K e^{lz} \frac{\partial^K}{\partial t^K},\ \square\square = \square_0' e^{lz} \tag{10}$$

Where $\square_0$, $\square_0$, $\square'_0$, $\square'_0$, $\square_0$, $\square'_0$ and m, *l* are constants and $\square_K$, $\square_K$ (K = 0, 1, 2, ... n) are the parameters associated with Kth order visco-elasticity.
Subsituting eq (9) and (10) in eqs (6), (7), (8), we get

$$(G_{\square} + G_{\square})\frac{\partial \Delta}{\partial x} + G_{\square}\square\square^2\ \square\square + mG_{\square}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \rho_0 \frac{\partial^2 u}{\partial t^2}, \tag{11}$$

$$G_{\square}\square\square^2\ \square\square + m\, G_{\square}\frac{\partial \nu}{\partial z} = \rho_0 \frac{\partial^2 \nu}{\partial t^2}, \tag{12}$$

$$(G_{\square} + G_{\square})\frac{\partial \Delta}{\partial z} + G_{\square}\square\square^2 w + \square\square G_{\square} m + 2G_{\square} m \frac{\partial w}{\partial z} = \rho_0 \frac{\partial^2 w}{\partial t^2}, \tag{13}$$

Where $G_{\square}\square = \sum_{K=0}^{n} \lambda_K \frac{\partial^K}{\partial t^K}$, $G_{\square}\square = \sum_{K=0}^{n} \mu_K \frac{\partial^K}{\partial t^K}$ and

$$\square^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \tag{14}$$

To investigate the surface wave propagation along the direction of Ox, we introduce the displacement potential □□(x, z, t) and □□(x, z, t) which are related to the displacement components as follows :

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z},\ w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{15}$$

Substituting eq (15) in eqs (11), (12), (13), we get

$$G_T \,\square^2 \,\square\square + mG_S \left(2 \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x} \right) = \frac{\partial^2\phi}{\partial t^2}, \tag{16}$$

$$G_S \,\square^2 \,\square\square + mG_S \,\frac{\partial\nu}{\partial z} = \frac{\partial^2\nu}{\partial t^2}, \tag{17}$$

$$G_S \,\square^2 \square\square + mG_P \,\frac{\partial\phi}{\partial x} + 2m\, G_S \,\frac{\partial\psi}{\partial z} = \frac{\partial^2\psi}{\partial t^2}. \tag{18}$$

Where,

$$U^2{}_{KT} = \frac{\lambda_K + 2\mu_K}{\rho_0}, U^2{}_{KS} = \frac{U_K}{\rho_0}, U^2{}_{KP} = \frac{\lambda_K}{\rho_0}, \tag{19}$$

&

$$G_T = \sum_{K=0}^{n} U_{KT}^2 \frac{\partial^K}{\partial t^K}, G_S = \sum_{K=0}^{n} U_{KS}^2 \frac{\partial^K}{\partial t^K}, G_P = \sum_{K=0}^{n} U_{KP}^2 \frac{\partial^K}{\partial t^K} \tag{20}$$

and similar relations in medium M_2 can be formed out by replacing $\square_K$, $\square_K$, $\square_0$ by $\square'_K$, $\square'_K$, $\square'_0$ and so on.

3. Solution of the problem

We seek the solution of (16), (17), (18) in the following forms

$$(\square, \square, \square) = [f(z), g(z), h(z)]\, e^{i\square\square(x-ct)} \tag{21}$$

Using eqs. (16)-(18) and (19)-(20), we get a set of differential equations for the medium M_1 as follows :

$$\frac{d^2 f}{dz^2} + 2m\, f_1{}^2 \frac{df}{dz} + h_1{}^2 f + i\,\square\square m\, f_1{}^2 g = 0, \tag{22}$$

$$\frac{d^2 h}{dz^2} + m \frac{dh}{dz} + K_1{}^2 h = 0, \tag{23}$$

$$\frac{d^2 g}{dz^2} + 2m \frac{dg}{dz} + K_1{}^2 g + i\,\square\, m\, l_1{}^2 f = 0. \tag{24}$$

Where

$$f_1{}^2 = \frac{\sum_{K=0}^{n} U_{KS}^2 (-i\alpha c)^K}{\sum_{K=0}^{n} U_{KT}^2 (-i\alpha c)^K},$$

$$h_1{}^2 = \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U_{KT}^2 (-i\alpha c)^K} - \square^2,$$

$$K_1{}^2 = \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U_{KS}^2 (-i\alpha c)^K} - \square^2,$$

$$l_1{}^2 = \frac{\sum_{K=0}^{n} U_{KP}^2 (-i\alpha c)^K}{\sum_{K=0}^{n} U_{KS}^2 (-i\alpha c)^K}.$$

And those for the medium M_2 are given by

$$\frac{d^2 f}{dz^2} + 2n f_1'^2 \frac{df}{dz} + h_1'^2 f + i\ □□m\ f_1'^2\ g = 0 \tag{25}$$

$$\frac{d^2 h}{dz^2} + l\ \frac{dh}{dz} + K_1'^2\ h = 0 \tag{26}$$

$$\frac{d^2 g}{dz^2} + 2\ l\ \frac{dg}{dz} + K_1'^2\ g + i\ □□l\ l_1'^2\ f = 0 \tag{27}$$

Where

$$f_1'^2 = \frac{\sum_{K=0}^{n} U'^2_{KS}\left(-i\alpha c\right)^K}{\sum_{K=0}^{n} U'^2_{KT}\left(-i\alpha c\right)^K},$$

$$h_1'^2 = \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U'^2_{KT}\left(-i\alpha c\right)^K} - \alpha^2,$$

$$K_1'^2 = \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U'^2_{KS}\left(-i\alpha c\right)^K} - \alpha^2,$$

$$l_1'^2 = \frac{\sum_{K=0}^{n} U'^2_{KP}\left(-i\alpha c\right)^K}{\sum_{K=0}^{n} U'^2_{KS}\left(-i\alpha c\right)^K}.$$

Equations (22)-(24) and (25)-(27) must have exponential solutions in order that f, g, h will describes surface waves, they must become vanishingly small as z □□□.

Hence for the medium M_1

$$□□(x, z, t) = \left\{A\, e^{-\lambda_1 z} + B\, e^{-\lambda_2 z}\right\} e^{i□□(x-ct)} \tag{28}$$

$$□□(x, z, t) = \left\{C\, e^{-\lambda_1 z} + D\, e^{-\lambda_2 z}\right\} e^{i□□(x-ct)} \tag{29}$$

$$□□(x, z, t) = E\, e^{-\lambda_3 z + i\lambda(x-ct)} \tag{30}$$

and similarly for the medium M_2 are given by

$$□□(x, z, t) = \left\{A'\, e^{-\lambda'_1 z} + B'\, e^{-\lambda'_2 z}\right\} e^{i□□(x-ct)} \tag{31}$$

$$□□(x, z, t) = \left\{C'\, e^{-\lambda'_1 z} + D'\, e^{-\lambda'_2 z}\right\} e^{i□□(x-ct)} \tag{32}$$

$$□□(x, z, t) = E'\, e^{-\lambda'_3 z + i\alpha(x-ct)} \tag{33}$$

Where $□_j$ and $□'_j$ (j = 1, 2) are the roots of the equations

$$\lambda^4 + 2m\left[1 + \frac{\sum_{K=0}^{n} U^2_{KS}\left(-i\alpha c\right)^K}{\sum_{K=0}^{n} U^2_{KT}\left(-i\alpha c\right)^K}\right]\lambda^3 + \left[\frac{\alpha^2 c^2}{\sum_{K=0}^{n} U^2_{KS}\left(-i\alpha c\right)^K} - \alpha^2 + 4m^2 \frac{\sum_{K=0}^{n} U^2_{KS}\left(-i\alpha c\right)^K}{\sum_{K=0}^{n} U^2_{KT}\left(-i\alpha c\right)^K}\, \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U^2_{KT}\left(-i\alpha c\right)^K} - \alpha^2\right]\lambda^2 +$$

$$\left[\frac{\sum_{K=0}^{n} U^2_{KS}\left(-i\alpha c\right)^K}{\sum_{K=0}^{n} U^2_{KT}\left(-i\alpha c\right)^K}\left\{\frac{\alpha^2 c^2}{\sum_{K=0}^{n} U^2_{KS}\left(-i\alpha c\right)^K} - \alpha^2\right\} + \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U^2_{KT}\left(-i\alpha c\right)^K} - \alpha^2\right] 2m\lambda + \left[\left\{\frac{\alpha^2 c^2}{\sum_{K=0}^{n} U^2_{KT}\left(-i\alpha c\right)^K} - \alpha^2\right\}\left\{\frac{\alpha^2 c^2}{\sum_{K=0}^{n} U^2_{KS}\left(-i\alpha c\right)^K} - \alpha^2\right\} + \alpha^2 m^2 \frac{\sum_{K=0}^{n} U^2_{KP}\left(-i\alpha c\right)^K}{\sum_{K=0}^{n} U^2_{KT}\left(-i\alpha c\right)^K}\right] = 0 \quad (34)\ (16)\ \&$$

$$\lambda'^4+2m\left[1+\frac{\sum_{K=0}^{n}U'^2_{KS}(-i\alpha c)^K}{\sum_{K=0}^{n}U'^2_{KT}(-i\alpha c)^K}\right]\lambda'^3+$$

$$\left[\frac{\alpha^2c^2}{\sum_{K=0}^{n}U'^2_{KS}(-i\alpha c)^K}+4l^2\frac{\sum_{K=0}^{n}U'^2_{KS}(-i\alpha c)^K}{\sum_{K=0}^{n}U'^2_{KT}(-i\alpha c)^K}-2\alpha^2+\frac{\alpha^2c^2}{\sum_{K=0}^{n}U'^2_{KT}(-i\alpha c)^K}\right]\lambda'^2$$

$$+\left[\frac{\sum_{K=0}^{n}U'^2_{KS}(-i\alpha c)^K}{\sum_{K=0}^{n}U'^2_{KT}(-i\alpha c)^K}\left\{\frac{\alpha^2c^2}{\sum_{K=0}^{n}U'^2_{KS}(-i\alpha c)^K}-\alpha^2\right\}+\frac{\alpha^2c^2}{\sum_{K=0}^{n}U'^2_{KT}(-i\alpha c)^K}-\lambda^2\right]2l\lambda'$$

$$+\left[\left\{\frac{\alpha^2c^2}{\sum_{K=0}^{n}U'^2_{KT}(-i\alpha c)^K}-\alpha^2\right\}\left\{\frac{\alpha^2c^2}{\sum_{K=0}^{n}U'^2_{KS}(-i\alpha c)^K}-\alpha^2\right\}+\alpha^2l^2\frac{\sum_{K=0}^{n}U'^2_{KP}(-i\alpha c)^K}{\sum_{K=0}^{n}U'^2_{KT}(-i\alpha c)^K}\right]=0 \qquad (35)\ \&\ (17)$$

For the media M_1 and M_2 respectively, we take into considering the real roots of eq (34) and eq (35). The constants A, B and A', B' are related with C, D and C', D' in eqs (28)-(30) and (31)-(33) by means of first equations in (22)-(24) and (25)-(27)

Equating the co-efficients of $e^{-\lambda_1 z}, e^{-\lambda_2 z}, e^{-\lambda'_1 z}, e^{-\lambda'_2 z}$ to zero after substituting eqs (28)-(30) and (31)-(33) in the first eqs in (22)-(24) and (25)-(27)respectively, we get

C = $□_1$ A ; D = $□_2$ B, C' = $□'_1$A', D' = $□'_2$ B'

Where

$$□_j=\frac{i}{\alpha m f_1^2}\left[□_j^2-2mf_1^2\,□_j+h_1^2\right] \qquad (36)$$

$$□_j'=\frac{i}{\alpha l f_1'^2}\left[□_j'^2-2n\,f_1'^2\,□'_j+h_1'^2\right] \qquad (37)\ \&\ (18)$$

4. Boundary Conditions

(i) The displacement components at the boundary surface between the media M_1 and M_2 must be continuous at all times and positions.

i.e. [u, □, w] M_1 = [u, □, w] M_2 at z = 0

(ii) The stress components $□_{31}$, $□_{32}$ and $□_{33}$ must be continuous at the boundary z = 0.

i.e. [$□_{31}$, $□_{32}$, $□_{33}$] M_1= [$□_{31}$,$□_{32}$,$□_{33}$]M_2 at z = 0

Where

$$□31= D\mu\left(2\frac{\partial^2\phi}{\partial x\partial z}+\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\right), \qquad (38)$$

$$□32= D\mu\frac{\partial v}{\partial z}, \qquad (39)$$

$$□33= D_\lambda\nabla^2\phi+2D_\mu\left(\frac{\partial^2\phi}{\partial z^2}+\frac{\partial^2\psi}{\partial x\partial z}\right) \qquad (40)$$

Applying the boundary conditions, we have

$$A(1-i□_1□_1)+B(1-i□_2□_2)+A'(i□'_1□'_1-1)+B'(i□'_2□'_2-1)=0 \qquad (41)$$

$$E= E' \qquad (42)$$

$$A(□_1+i□_1)+B(□_2+i□_2)+A'(-□'-i□'_1)+B'(-□'_2-i□'_2)=0 \qquad (43)$$

$$□^*_K[(2i□_1+□_1+□_1^2□_1)A+(2i□_2+□_2+□_2^2□_2)B]=\mu'^*_K[(2i□'_1+□'_1+□_1'^2□'_1)A'+(2i□'_2+□'_2+□_2'^2□'_2)B']$$

(44)

$\square^*{}_K[-\square_3\, E] \quad = \mu'^*_K[-\square'_3\, E']$ (45)

A $[\square^*{}_K(\square_1{}^2 - 1) + 2\square^*{}_K(\square_1{}^2 - i\,\square_1\,\square_1)]$ + B $[\square^*{}_K(\square_2{}^2 - i\,\square_2\,\square_2)]$= A' $[\square'^*{}_K(\square_1{}'^2 - 1) + 2\square'^*{}_K(\square_1{}'^2 - i\,\square'_1\,\square'_1)]$ + B' $[\square'^*{}_K(B_2{}'^2 - 1) + 2\,\square'^*{}_K(\square_2{}'^2 - i\,\square'_2\,\square'_2)]$ (46)

Where,

$\square_j = \dfrac{\lambda_j}{\alpha}$, $\square'_j = \dfrac{\lambda_j{}'}{\alpha}$, j = 1, 2. and

$\square^*{}_K = \sum_{K=0}^{n} \lambda_K(-i\alpha c)^K$, $\square^*{}_K = \sum_{K=0}^{n} \mu_K(-i\alpha c)^K$

$\square'^*{}_K = \sum_{K=0}^{n} \lambda'_K(-i\alpha c)^K$, $\square'^*{}_K = \sum_{K=0}^{n} \mu'_K(-i\alpha c)^K$.

From eqs (42) and (43), we have E = E' = 0. Thus there is no propagation of displacement $\square$. Hence SH-waves do not occur in this case.

Finally, eliminating the constants A, B, E, E' from eqs. (41), (43), (44) and (46), we get

det (a_{ij})= 0 i, j = 1, 2, 3, 4 (47)

where,

$a_{21} = \square_1 + i\,\square_1$,

$a_{22} = \square_2 + i\,\square_2$,

$a_{23} = -\square'_1 - i\,\square'_1$,

$a_{24} = -\square'_2 - i\,\square'_2$,

$a_{11} = (1 - i\,\square_1\,\square_1)$,

$a_{12} = 1 - i\,\square_2\,\square_2$,

$a_{13} = (i\,\square'_1\,\square'_1 - 1)$,

$a_{14} = (i\,\square'_2\,\square'_2 - 1)$,

$a_{31} = \square'_K\,[2i\,\square_1 + \square_1(1 + \square_1{}^2)]$;

$a_{32} = [2i\,\square_2 + \square_2(1 + \square_2{}^2)]\,\square^*{}_K$,

$a_{33} = [2i\,\square'_1 + \square'_1(1 + \square_1{}'^2)]\,(-\square'^*{}_K)$,

$a_{34} = -[2i\,\square'_2 + \square_2{}'(1 + \square_2{}'^2)]\,\square'^*{}_K$,

$a_{41} = \square^*{}_K(\square_2{}' - 1) + 2\,\square^*{}_K(\square_1{}^2 - i\,\square_1\square_1)$,

$a_{42} = \square^*{}_K(\square_2{}^2 - 1) + 2\square^*{}_K(\square_2{}^2 - i\,\square_2\square_2)$,

$a_{43} = \square'^*{}_K(1 - \square_1{}'^2) + 2\square'^*{}_K(i\,\square_1{}'^*\,\square'_1 - \square_1{}'^2)$,

$a_{44} = \square'^*{}_K(1 - \square_2{}'^2) + 2\square'^*{}_K(i\,\square'_2\,\square'_2 - \square_2{}'^2)$.

From this equation (47), we get the velocity of surface waves in common boundary between two viscoelastic, non-homogeneous solid media of Voigt type, where the viscosity is of general nth order involving time rate of change of strain.

5. Particular Cases

Stonelay Waves It is the generalized form of Rayleigh waves in which we assume that the waves are propagated along the common boundary of two semi-infinite media M_1 and M_2. Therefore eq (47) determine the wave velocity equation for Stoneley waves in the case of general viscoelastic, non-homogeneous solid media of nth order involving time rate of strain.

Clearly from eq (47), it is follows that wave velocity of the Stonelay waves depends upon the non-homogeneity of the material medium and the viscosity.

In case of absence of non-homogeneity and viscoelastic medium of 1st order involving time rate of change of strain is taken. Then equation (47) reduces to,

$|bij| = 0$, i, j = 1, 2, 3, 4...... (*)

Where,

$b_{11} = T_1$, $b_{12} = T'_1$, $b_{13} = -1$, $b_{14} = 1$,

$b_{21} = 1$, $b_{22} = -1$, $b_{23} = T_2$, $b_{24} = T'_2$,

$b_{31} = (\mu_0 - i\omega c\mu_1)(T_1^2 - 1)$, $b_{32} = (\mu_0' - i\omega c\mu_1')(T_1'^2 - 1)$,

$b_{33} = (\mu_0 - i\omega c\mu_1)T_2$, $b_{34} = -(\mu_0 - i\omega c\mu_1)T_2'$,

$$b_{41} = (\mu_0 - i\omega c\mu_1)T_1,\ b_{42} = (\mu_0^{/} - i\omega c\mu_1^{/})T_1^{/},$$
$$b_{43} = (\lambda_0 - i\omega c\lambda)(1+T_2^2) + 2(\mu_0 - \mu_1(i\omega c)T_2^2,$$
$$b_{44} = -(\lambda_0 - i\omega c\lambda_1)(1+T_2^{/2}) + 2(\mu_0^{/} - \mu_1^{/}(i\omega c)T_2^{/2},$$

Here,

$$T_1^2 = \left[\frac{\rho_0 c^2}{\mu_0 - i\omega c\mu_1} - 1\right], T_2^2 = \left[\frac{\rho_0 c^2}{(\lambda_0 + 2\mu_0) - i\omega c(\lambda_1 + 2\mu_1)} - 1\right] \ldots\ldots.., (**)$$

$$T_1^{/2} = \left[\frac{\rho_0^{/} c^2}{\mu_0^{/} - i\omega c\mu_1^{/}} - 1\right], T_2^2 = \left[\frac{\rho_0^{/} c^2}{(\lambda_0^{/} + 2\mu_0^{/}) - i\omega c(\lambda_1^{/} + 2\mu_1^{/})} - 1\right] \ldots\ldots..,$$

Equation (*) gives the wave velocity equation of Stoneley waves in a viscoelastic media of Voigt type where the viscosity is of 1st order involving time rate of change of strain which is completely agreement with classical result given by P.R. Sengupta et al.

Further equation (47), of course, is in complete agreement with the corresponding classical result, when the effect of viscosity and non-homogeneity are ignored.

Rayleigh Waves To investigate the possibility of Rayleigh waves in a viscoelastic, non-homogeneous elastic media, we replace medium M_2 by vacuum, in the preceding problem.

We note also that SH-wave do not occur in this case.
Thus eqs. (44) and (46), reduces to

$$(2i\,\square_1 + \square_1 + \square_1^2\,\square_1)\,A + (2i\,\square_2 + \square_2 + \square_2^2\,\square_2)\,B = 0 \qquad (48)$$
$$[\square^*_K(\square_1^2 - 1) + 2\square^*_K(\square_1^2 - i\,\square_1\,\square_1)]\,A + [\square^*_K(\square_2^2 - 1) + 2\square^*_K(\square_2^2 - i\,\square_2\,\square_2)]\,B = 0 \qquad (49)$$

Eliminating A and B from eqs. (48) and (49), we have

$$[2i\,\square_1 + \square_1(\square_1^2 + 1)]\,[\square^*_K(\square_2^2 - 1 + 2\square^*_K(\square_2^2 - i\,\square_2\,\square_2)] - [2i\,\square_2 + \square_2(\square_2^2 + 1)][\square^*_K(\square_1^2 - 1) + 2\square^*_K(\square_1^2 - i\,\square_1\,\square_1)] = 0 \quad \ldots\ldots\ldots.. \quad (50)$$

Equation (50) gives wave velocity equation for Rayleigh waves in a non-homogeneous, viscoelastic solid medium of nth order involving time rate of strain.

Also, from eq (50) we see that Rayleigh waves depends on the viscosity and the non-homogeneity of the material medium.
In the absence of non-homogeneity & considering the viscoelastic media of 1st order including time rate of strain. equation (50) reduces to,

$$(T_1^2 - 1)[(\lambda_0 - i\omega c\lambda_1)(1+T_2^2) + 2(\mu_0 - \mu_1 i\omega c)T_2^2] + 4(\mu_0 - i\omega c\mu_1)T_2T_2 = 0 \qquad (51)$$

Where T_{1}, T_{2} are given by (**)
Thus, equation (51) gives the wave velocity equation of Rayleigh waves in a homogeneous general viscoelastic media of 1st order including time rate of strain, which tallies with corresponding classical result of P.R. Sengupta et. al.
For elastic media $\lambda_K = \lambda_K^{/},\quad \mu_K = \mu_K^{/} = 0,$,
Where K=1,2……n
Equation (50) becomes

$$[2i\,\square_1 + \square_1(\square_1^2 + 1)]\,[C_1^2(1 - \square_2^2) - 2C_2^2(1 - i\,\square_2\,\square_2] - [2i\,\square_2 + \square_2(\square_2^2 + 1)][C_1^2(\square_1^2 - 1) + 2C_2^2(\square_1^2 - i\,\square_1\,\square_1)] = 0 \quad \ldots(52)$$

Where,

$$C_1^2 = \frac{\lambda_0 + 2\mu_0}{\rho_0}\ ,\ C_2^2 = \frac{\mu_0}{\rho_0}.$$

. (52) determines the wave velocity equation of Rayleigh waves in a non-homogeneous isotropic elastic medium and this equation is in complete agreement with the corresponding classical result of Das et al.
Further when non-homogeneity of the material medium is neglected,
Eq. (52) becomes,

$$\sqrt[4]{\left[1 - \frac{C^2}{C_1^2}\right]\left[1 - \frac{C^2}{C_2^2}\right]} = \left[1 - \frac{C^2}{C_2^2}\right]^2 \qquad (53)$$

Which is in complete agreement with classical result of Bullen.

The equation (50), of course, is in agreement with the corresponding classical result given by Bullen,

Love Waves To investigate the possibility of love waves in a non-homogeneous, viscoelastic solid media, we replace medium M_2 is obtained by two horizontal plane surfaces at a distance H-apart, while M_1 remains infinite.
For medium M_1, the displacement component □□remains same as in general case given by eq. (14).
For the medium M_2, we pressure the full solution, since the displacement component along y-axis i.e. $□_2$ no longer diminishes with increasing distance from the boundary surface of two media.

Thus □'= $E_1 e^{\lambda'_3 z + i\alpha(x-ct)} + E_2 e^{-\lambda'_3 z + i\alpha(x-ct)}$ (54)

In this case, the boundary conditions are

(i) □□and $□_{32}$ are continuous at z = 0

(ii) $□_{32}$ = 0 at z = –H

applying boundary conditions (i) and (ii) and using eqs (31)-(33) , (38)-(40) and eq (54), we get

$$E = E_1 + E_2 \tag{55}$$

$$-□_3\, □^*_K\, E = □'^*_K\, [□'_3 E_1 - □'_3 E_2] \tag{56}$$

$$□'_3 E_1\, e^{\lambda'_3 H} - \lambda'_3 E_2\, e^{-\lambda'_3 H} = 0 \tag{57}$$

On eliminating the constant E, E_1 and E_2 from eqs (55), (56) and (57), we get

$$\tan(i\, □'_3 H) = \frac{i\lambda_3\, \mu^*_K}{\lambda'_3\, \mu'^*_K} \tag{58}$$

Thus equation (58) gives the wave velocity equation for Love waves in a non-homogeneous, viscoelastic elastic solid medium of nth order involving time rate of strain.
In the absence of non-homogeneity & considering the viscoelastic media of 1st order including time rate of strain.
Equation (58) reduces to,

$$T_1^{/}\left(\mu_0^{/} - i\omega c\mu_1^{/}\right)\tan\left(\omega T_1^{/} H\right) + i\left(\mu_0 - i\omega c\mu_1\right)T_1 = 0$$

which gives the dispersion equation of Love waves in a viscoelastic solid medium of 1st order involving time rate of strain, which is in well agreement with the corresponding classical result given by P.R. Sengupta et. al.

For elastic media $\lambda_K = \lambda_K^{/}, \quad \mu_K = \mu_K^{/} = 0,$ (59)

(Where K=1,2……n) with absence of non-homogeneity of material medium, equation (59) is in complete agreement with corresponding classical result of Bullen.
The equation (58), of course, is in agreement with the corresponding classical result given by Bullen.

Conclusions

I. Rayleigh waves in a non-homogeneous, general viscoelastic solid medium of higher order of Voigt type, we find that the wave velocity equation proves that there is dispersion of waves due to the presence of non-homogeneity and viscosity. The results are in complete agreement with the corresponding classical results when non-homogeneity and viscous field are neglected.

II. Love waves in a non-homogeneous, general viscoelastic solid medium of higher order of Voigt type; we find that the wave velocity equation proves that there is dispersion of waves due to the presence of non-homogeneity and viscosity. The results are in complete agreement with the corresponding classical results when non-homogeneity and viscous field are neglected

III. Wave velocity equation of Stoneley waves is very similar to the corresponding problem in the classical theory of elasticity. Here also there is dispersion of waves due to the presence of non-homogeneity and viscoelastic nature of the solid.

IV. The solution of wave velocity equation for Stoneley waves cannot be determined by easy analytical methods however we can apply numerical technique to solve this determinantal equation by choosing suitable values of physical constants for both media M1 and M2.

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