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RESEARCH ARTICLE

K-TRIPOTENT OF POWER SYMMETRIC MATRICES

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ABSTRACT

In this paper, the concept of k-tripotentof power symmetric matrices is introduced. Conditions for power symmetric matrices to be k-tripotent are discussed.

INTRODUCTION

Ann Lee (1976) has initiated the study of secondary symmetric matrices. Hill and Waters (1992) have developed a theory of k-real and k-hermitian matrices as a generalization of secondary real and secondary hermitian matrices. Krishnamoorthy, Gunasekaran and Bhuvaneswari (2011) have studied the elementary properties of symmetric and k-symmetric matrices. Krishnamoorthy and Meenakshi (2013) have studied the basic concepts of k-tripotent matrices as generalization of tripotent matrices. Throughout the paper, let C^n denote the unitary space of order n and C_{nxn} be the space all complex n x n matrices. For a matrices $A \epsilon C_{nxn}$ $\bar{A}$, A^T, $A^*$and A^{-1} denote conjugate, transpose, conjugate transpose and inverse of the matrix A respectively. Let 'k' be a fixed product of disjoint transposition in S_n- the set of all permutation on {1, 2 ....n}. Hence it is involutory (that is K^2 = identity permutation).If 'K' is the associated permutation matrix of 'k' then it clearly satisfies the following properties:

$K^2 = 1$ and $K = K^T = \bar{K} = K^*$

A matrix A = (a_{ij}) in C_{nxn} is said to be k-tripotent, if $a_{ij} = \sum_{t=1}^{n} a_{k(i)t} \left[\sum_{m=1}^{n} a_{tm} a_{mk(j)} \right]$ this is equivalent $A=KA^3K$.

k-tripotent of Power Symmetric Matrices

Definition 2.1

A Symmetric matrix is a square matrix which is equal to its transpose. (ie) $A = A^T$.

Example 2.2

$$A=\begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & -1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix} \quad A^T=\begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & -1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

Definition 2.3

A square matrix is said to be a k-symmetric,if $A = KA^TK$.

Definition 2.4

A cube symmetric matrix is a square matrix whose cube is equal to its transpose. (i.e) $A^3 = A^T$.

Definition 2.5

A square matrix is said to be a k-cube symmetric,if $A^T = KA^3K$.

Theorem 2.6

If $A \in C^{nxn}$ then any two of the following imply the other one
(a)A is tripotent
(b)A is symmetric
(c)A is cube symmetric

Proof

(a)and (b) $\Rightarrow$ (c)
$A=A^3$ and $A = A^T$ $\Rightarrow A^3= A^T$, Hence A is cube symmetric.

(b)and (c) $\Rightarrow$ (a)
$A=A^T$ and $A^3=A^T$ $\Rightarrow$ $A= A^3$, Hence A is tripotent.

(c)and (a) $\Rightarrow$ (b)
$A^3=A^T$ and $A=A^3$ $\Rightarrow$ $A^T = A$, Hence A is Symmetric.

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Theorem 2.7

If $A \in C^{nxn}$ then any two of the following imply the other one

(a) A is k-tripotent
(b)A is k-symmetric
(c)A is cube symmetric

Proof

(a)and (b) $\Rightarrow$ (c)
(b)
$KA^3K=A$ and $KA^TK=A$
$KA^3K = KA^TK \Rightarrow A^3=A^T$, Hence A is cube symmetric.

(b)and (c) $\Rightarrow$ (a)
$KA^TK=A$ and $A^3 =A^T$
Substitute $A^3=A^T$ in $KA^TK =A \Rightarrow KA^3K=A$, Hence A is k-tripotent.

(c)and (a) $\Rightarrow$ (b)
$A^3=A^T$ and $KA^3K=A$
Substitute $A^3=A^T$ in $KA^3K=A \Rightarrow KA^TK =A$, Hence A is k-symmetric.

Theorem 2.8

If $A \in C^{nxn}$ then any two of the following imply the other one

(a) A is k-tripotent
(b) A is symmetric
(c) A is k-cube symmetric

Proof

(a)and (b) $\Rightarrow$ (c)
$KA^3K=A$ and $A=A^T \Rightarrow KA^3K =A^T$, Hence A is k-cube symmetric.

(b)and (c) $\Rightarrow$ (a)
$A=A^T$ and $KA^3K =A^T \Rightarrow KA^3K=A$, Hence A is k-tripotent.

(c)and (a) $\Rightarrow$ (b)
$KA^3K=A^T$ and $KA^3K =A \Rightarrow A^T=A$, Hence A is symmetric.

Theorem 2.9

Let KA be a k-tripotent matrix then the following are equivalent:
(a)KA is symmetric
(b)KA is k-symmetric
(c)KA is cube symmetric.

Proof

(a) $\Rightarrow$ (b)
$(KA)^T =KA \Rightarrow (KA)^T=(KA)^3$ (by Remark 2.4 [4])
pre and post multiply by K, $K (KA)^TK = K(KA)^3K=KA$,
Hence KA is k-symmetric.

(b) $\Rightarrow$ (c)
$K(KA)^TK =KA$
pre and post multiply by K, $(KA)^T =K(KA)K=(KA)^3$
Hence KA is cube symmetric.

(c) $\Rightarrow$ (a)
$(KA)^T=(KA)^3= KA$ (by Remark 2.4[4])
Hence KA is symmetric.

Theorem 2.10

Let A be a k-tripotent matrix then the following are equivalent:

(a) KA is cube symmetric
(b) KA is symmetric
(c) A is cube symmetric

Proof

(a) $\Rightarrow$ (b)
$(KA)^3 =(KA)^T \Rightarrow (KA)= (KA)^T$ (by Remark 2.4[4])
Hence KA is symmetric.

(b)$\Rightarrow$ (c)
$(KA)^T=KA \Rightarrow A^TK=A^3K$ (by Remark 2.4[4])
post multiply by K, $A^T = A^3$, Hence A is cube symmetric.

(c) $\Rightarrow$ (a)
$A^3=A^T$, but $(KA)^3 =KA=A^3K=A^TK$
$(KA)^3 =(KA)^T$, Hence (KA) is cube symmetric.

Theorem 2.11

Let A be a k-tripotent matrix then the following are equivalent:

(a)KA is k-cube symmetric
(b)A is symmetric
(c)A is k-cube symmetric
(d)KA is k-symmetric.

Proof

(a) $\Rightarrow$ (b)
$K(KA)^3K =(KA)^T=A^TK$
$AK =A^TK \Rightarrow A=A^T$, Hence A is symmetric.

(b) $\Rightarrow$ (c)
$A =A^T \Rightarrow KA^3K =A^T$, Hence A is k-cube symmetric.

(c) $\Rightarrow$(d)
$KA^3K =A^T$
Post mulitiply by K, $KA^3=A^TK=(KA)^T$
Pre and post multiply by K, $A^3K=K(KA)^TK \Rightarrow KA=K(KA)^T K$
Hence KA is k-symmetric.

(d) $\Rightarrow$(a)
$KA =K (KA)^TK$,but $(KA)^3 =KA$ (by Remark 2.4[4])
$(KA)^3=K (KA)^TK$
Pre and post multiply K, $K (KA)^3K=(KA)^T$
Hence KA is k-cube symmetric.

REFERENCES

Ann Lee, *secondary symmetric, secondary skew symmetric, secondary orthogonal matrices;* Period Math Hungary, 7 (1976) 63-76.
Hill R.D and Waters S.R, *On k- real and k-hermitian matrices;* Linear Alg. Appl.169(1992) 17-29.
Krishnamoorthy.S, Gunasekaran.K and Bhuvaneswari. G, *On k-orthogonal Matrices;* International J. of Math.Sci&Engg.Appls (IJMSEA) ISSN 0973-9424, Vol.5 No V (Sep 2011) PP 235 -243.
Krishnamoorthy.S and Meenakshi.P.S, *On k-tripotent matrices;* International J. of Math.Sci&Engg.Appls (IJMSEA) ISSN 0973-9424 Vol.7 No.1 (January 2013) PP101-105.

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